Universal Quantum Computation with Gapped Boundaries

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Refs.: C, Cheng, Wang, *Comm. Math. Phys.* (2017) 355: 645.
C, Cheng, Wang, *Phys. Rev. Lett.* **119**, 170504 (2017).

Introduction: Quantum Computation

- Quantum computing:
  - Feynman (1982): Use a quantum-mechanical computer to simulate quantum physics (classically intractable)
  - Encode information in qubits, gates = unitary operations
  - Applications: Factoring/breaking RSA (Shor, 1994), quantum machine learning, ...

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  - Encode information in qubits, gates = unitary operations
  - Applications: Factoring/breaking RSA (Shor, 1994), quantum machine learning, ...
- Major challenge: local decoherence of qubits

- **Topological** quantum computing (TQC) (Kitaev, 1997; Freedman et al., 2003):
  - Encode information in *topological* degrees of freedom
  - Perform *topologically protected* operations

Topological Quantum Computation

- Traditional realization of TQC: *anyons* in *topological phases of matter* (e.g. Fractional Quantum Hall - FQH)
  - Elementary (quasi-)particles in 2 dimensions s.t. $|\psi_1\psi_2\rangle = e^{i\phi}|\psi_2\psi_1\rangle$

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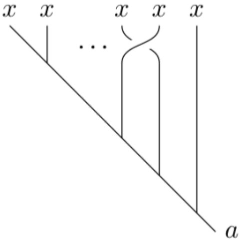
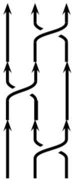
  - Elementary (quasi-)particles in 2 dimensions s.t. $|\psi_1\psi_2\rangle = e^{i\phi}|\psi_2\psi_1\rangle$
- Qubit encoding: degeneracy arising from the *fusion rules*
  - e.g. Toric code: $e^i m^j \otimes e^k m^l = e^{(i+k)(\text{mod } 2)} m^{(j+l)(\text{mod } 2)}$ $(i, j = 0, 1)$
  - e.g. Ising: $\sigma \otimes \sigma = 1 \oplus \psi$, $\psi \oplus \psi = 1$
  - e.g. Fibonacci: $\tau \otimes \tau = 1 \oplus \tau$

Topological Quantum Computation

- Topologically protected operations: Braiding of anyons
  - Move one anyon around another $\rightarrow$ pick up phase (due to Aharonov-Bohm)





Figures: (1) Z. Wang, *Topological Quantum Computation*.
(2) C. Nayak et al., *Non-abelian anyons and topological quantum computation.*

Problem

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- These are difficult to realize, existence is still uncertain
- Question: Given a top. phase that supports only abelian anyons, is it possible "engineer" other non-abelian objects?
  - Answer: Yes! We consider boundaries of the topological phase $\rightarrow$ *gapped boundaries*
  - We'll even get a universal gate set from gapped boundaries of an abelian phase (specifically, bilayer $\nu = 1/3$ FQH)

Overview

- Framework of TQC
- Introduce gapped boundaries and their framework
- Gapped boundaries for TQC
- Universal TQC with gapped boundaries in bilayer $\nu = 1/3$ FQH
- Summary and Outlook

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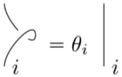
Framework of TQC

Formally, an *anyon model* $\mathcal{B}$ consists of the following data:

- Set of anyon types/labels: $\{a, b, c...\}$, one of which should represent the vacuum $\mathbf{1}$
  - Each anyon type has a topological twist $\theta_i \in \mathrm{U}(1)$:



[1] More general cases exist, but are not used in this talk.

Figures: Z. Wang, *Topological Quantum Computation* (2010)

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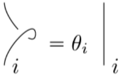
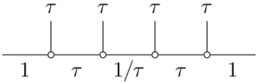


- For each pair of anyon types, a set of fusion rules: $a \otimes b = \oplus_c N_{ab}^c c$.
  - The fusion space of $a \otimes b$ is a vector space V_{ab} with basis V_{ab}^c.[1]
  - The fusion space of $a_1 \otimes a_2 \otimes ... \otimes a_n$ to b is a vector space $V_{a_1 a_2 \ldots a_n}^b$ with basis given by anyon labels in intermediate segments, e.g.



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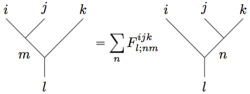
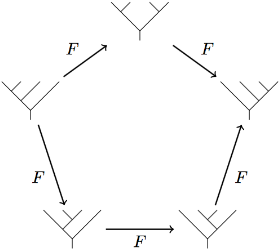
Formally, an *anyon model* $\mathcal{B}$ consists of the following data: [Cont'd]

- Associativity: for each (a, b, c, d), a set of (unitary) linear transformations $\{F^{abc}_{d;ef} : V^{abc}_d \to V^{abc}_d\}$ satisfying "pentagons"





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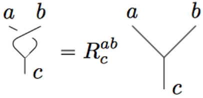
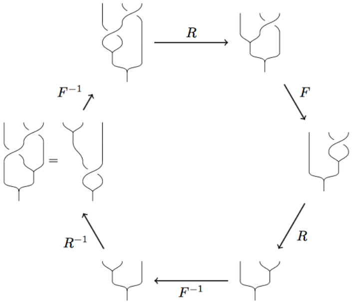
Framework of TQC

Formally, an *anyon model* $\mathcal{B}$ consists of the following data: [Cont'd]

- (Non-degenerate) braiding: For each (a, b, c) s.t. $N^c_{a,b} \neq 0$, a set of phases[1] $R^c_{ab} \in \mathrm{U}(1)$ compatible with associativity ("hexagons"):





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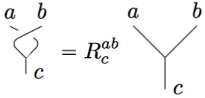

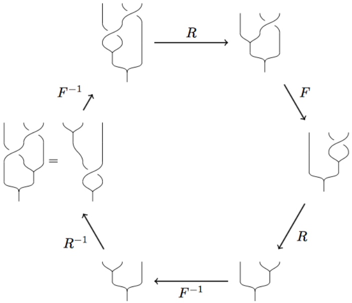

Mathematically, this is captured by a (unitary) *modular tensor category* (UMTC)

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Framework of TQC

- In this framework, a 2D *topological phase of matter* is an equivalence class of gapped Hamiltonians $\mathcal{H} = \{H\}$ whose low-energy excitations form the same anyon model $\mathcal{B}$
- Examples (on the lattice) include Kitaev's quantum double models, Levin-Wen string-net models, ...

Framework of TQC: Example

- Physical system: Bilayer FQH system, $1/3$ Laughlin state of opposite chirality in each layer
- Equivalent to $\mathbb{Z}_3$ toric code (Kitaev, 2003)
- Anyon types: $e^a m^b$, $a, b = 0, 1, 2$
- Twist: $\theta(e^a m^b) = \omega^{ab}$ where $\omega = e^{2\pi i/3}$
- Fusion rules: $e^a m^b \otimes e^c m^d \to e^{(a+c)(\text{mod } 3)} m^{(b+d)(\text{mod } 3)}$
- F symbols all trivial (0 or 1)
- $R_{e^a m^b, e^c m^d} = e^{2\pi i bc/3}$
- UMTC: $\text{SU}(3)_1 \times \overline{\text{SU}(3)_1} \cong \mathfrak{D}(\mathbb{Z}_3) = \mathcal{Z}(\text{Rep}(\mathbb{Z}_3)) = \mathcal{Z}(\text{Vec}_{\mathbb{Z}_3})$

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- Framework of TQC
- **Introduce gapped boundaries and their framework**
- Gapped boundaries for TQC
- Universal gate set with gapped boundaries in bilayer $\nu = 1/3$ FQH
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Gapped Boundaries: Framework

- A *gapped boundary* is an equivalence class of gapped local (commuting) extensions of $H \in \mathcal{H}$ to the boundary

Gapped Boundaries: Framework

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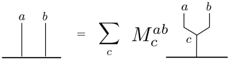
- In the anyon model: Collection of bulk bosonic ($\theta = 1$) anyons which condense to vacuum on the boundary (think Bose condensation)
  - All other bulk anyons condense to confined "boundary excitations" α, β, γ...
- Mathematically, Lagrangian algebra $\mathcal{A} \in \mathcal{B}$

Gapped Boundaries: Framework

More rigorously, gapped boundaries come with M symbols (like F symbols for the bulk):



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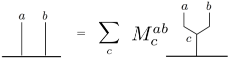

(More generally, we also define these with boundary excitations, but that is unnecessary for this talk.)

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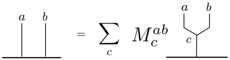

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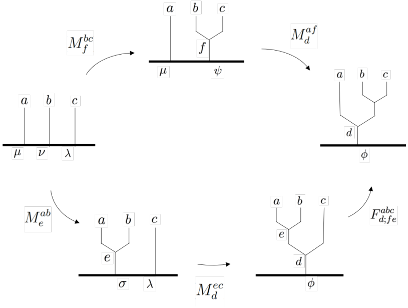
M symbols must be compatible with F symbols ("mixed pentagons"):



Gapped Boundaries: Example

- Physical system: Bilayer $\nu = 1/3$ FQH, equiv. to $\mathbb{Z}_3$ toric code
- Anyon types: $e^a m^b$, $a, b = 0, 1, 2$

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- Anyon types: $e^a m^b$, $a, b = 0, 1, 2$
- Two gapped boundary types:
  - Electric charge condensate: $\mathcal{A}_1 = 1 \oplus e \oplus e^2$
  - Magnetic flux condensate: $\mathcal{A}_2 = 1 \oplus m \oplus m^2$

Gapped Boundaries: Example

- Physical system: Bilayer $\nu = 1/3$ FQH, equiv. $\mathbb{Z}_3$ toric code
- We will work mainly with $\mathcal{A}_1 = 1 \oplus e \oplus e^2$:
  - Algebraically, $\mathcal{A}_1$, $\mathcal{A}_2$ are equivalent by electric-magnetic duality
  - Easier to work with charge condensate - read-out can be done by measuring electric charge (Barkeshli, 2016)
  - It is interesting to consider *both* $\mathcal{A}_1$ and $\mathcal{A}_2$ at the same time - we do this in a separate paper[3], will briefly mention in our Outlook

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- M symbols for this theory are all 0 or 1

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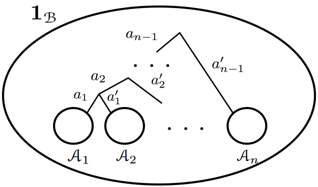
Gapped Boundaries for TQC

Qudit encoding:

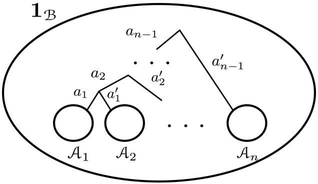
- Gapped boundaries give rise to a natural ground state degeneracy: *n* gapped boundaries on a plane, with total charge vacuum

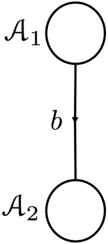


Gapped Boundaries for TQC

Qudit encoding:

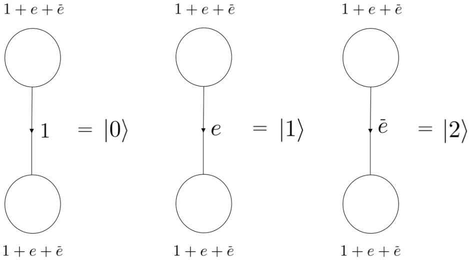
- Gapped boundaries give rise to a natural ground state degeneracy: n gapped boundaries on a plane, with total charge vacuum
- For qudit encoding: use $n = 2$





Gapped Boundaries for TQC: Example

For our bilayer $\nu = 1/3$ FQH system, we have:



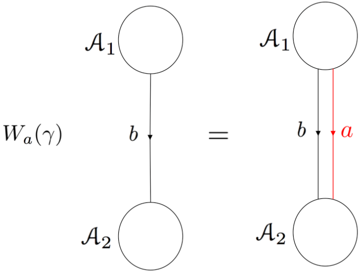
Gapped Boundaries for TQC

Topologically protected operations:

- Tunnel-*a* operations
- Loop-*a* operations
- Braiding gapped boundaries
- Topological charge measurement*

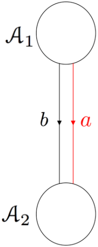
Tunnel-a Operations

Starting from state $|b\rangle$, tunnel an a anyon from $\mathcal{A}_1$ to $\mathcal{A}_2$:

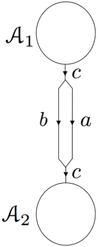


Tunnel-a Operations

Compute using M-symbols:



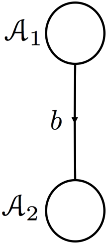
$$= \sum_c M_c^{ab}(\mathcal{A}_2)[M_c^{ab}(\mathcal{A}_1)]^\dagger$$



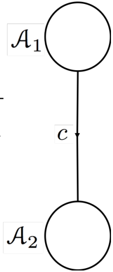
Tunnel-a Operations

Result:

$W_a(\gamma)$

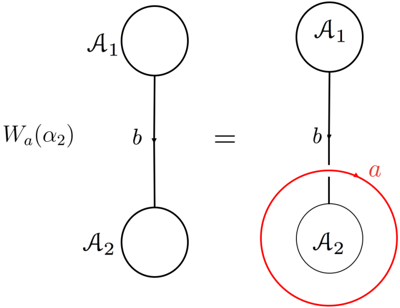


$$= \sum_c M_c^{ab}(\mathcal{A}_1)[M_c^{ab}]^\dagger(\mathcal{A}_2)\sqrt{\frac{d_a d_b}{d_c}} \quad (1)$$



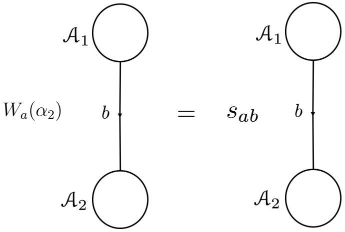
Loop-a Operations

Starting from state $|b\rangle$, loop an a anyon around one of the boundaries:

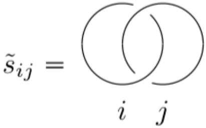


Loop-a Operations

Similar computation methods lead to the formula:

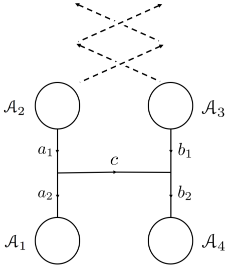


where $s_{ab} = \tilde{s}_{ab}/d_b$ is given by the modular S matrix of the theory:



Braiding Gapped Boundaries

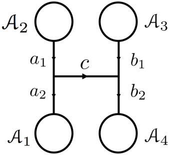
Braid gapped boundaries around each other:



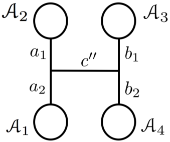
(Mathematically, this gives a representation of the (spherical) $2n$-strand pure braid group.)

Simplify with (bulk) R and F moves to get:

σ_2^2



$$= \sum_{c,c'} F^{a_2 a_1 b_1}_{b_2; c' c} R^{b_1 a_1}_{c'} R^{a_1 b_1}_{c} \left(F^{a_2 a_1 b_1}_{b_2}\right)^{-1}_{c'' c'}$$

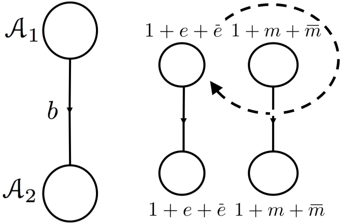


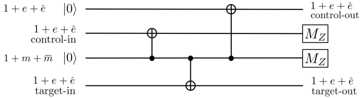
(2)

Gapped Boundaries for TQC: Example

For our bilayer $\nu = 1/3$ FQH case: ($\mathcal{A}_1 = \mathcal{A}_2 = 1 \oplus e \oplus e^2$)

- Tunnel an e anyon from $\mathcal{A}_1$ to $\mathcal{A}_2$: $W_a(\gamma)|b\rangle = |a \otimes b\rangle$
  $\rightarrow W_e(\gamma) = \sigma_3^x$, where $\sigma_3^x|i\rangle = |(i+1)(\mathrm{mod}\ 3)\rangle$
- Loop an m anyon around $\mathcal{A}_2$: $W_m(\alpha_2)|e^j\rangle = \omega^j|e^j\rangle$
  $\rightarrow W_m(\alpha_2) = \sigma_3^z$
- Braid gapped boundaries: get $\wedge\sigma_3^z$





Topological Charge Projection/Measurement

- Motivation:
  - Property F conjecture (Naidu and Rowell, 2011): Braidings alone cannot be universal for TQC for most physically plausible systems

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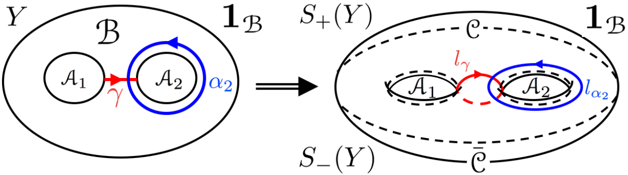
  - Property F conjecture (Naidu and Rowell, 2011): Braidings alone cannot be universal for TQC for most physically plausible systems
- Topological charge projection (TCP) (Barkeshli and Freedman, 2016):
  - Doubled theories: Wilson line lifts to a loop → measure topological charge through the loop



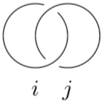
  - Resulting projection operator: ($\mathcal{O}_x(\beta) = W_{x\bar{x}}(\gamma_i)$ or $W_x(\alpha_i)$)

$$P_\beta^{(a)} = \sum_{x \in \mathcal{C}} S_{0a} S_{xa}^* \mathcal{O}_x(\beta). \tag{3}$$

Topological Charge Projection/Measurement

- Topological charge projection (TCP): [Cont'd]
  - Given an anyon theory $\mathcal{C}$, its $\mathcal{S}, \mathcal{T}$ matrices

$$\mathcal{S} = \left\{ \tilde{s}_{ij} = \right.$$



$$\left. \right\}, \qquad \mathcal{T} = \mathrm{diag}(\theta_i)$$

    give mapping class group representations $V_{\mathcal{C}}(Y)$ for surfaces Y.
  - Barkeshli and Freedman showed that topological charge projections generate all matrices in $V_{\mathcal{C}}(Y)$

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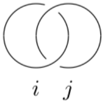
  - Given an anyon theory $\mathcal{C}$, its $\mathcal{S}, \mathcal{T}$ matrices

$$\mathcal{S} = \left\{ \tilde{s}_{ij} = \right.$$



$$\left. \right\}, \qquad \mathcal{T} = \mathrm{diag}(\theta_i)$$

    give mapping class group representations $V_{\mathcal{C}}(Y)$ for surfaces Y.
  - Barkeshli and Freedman showed that topological charge projections generate all matrices in $V_{\mathcal{C}}(Y)$
- General topological charge measurements (TCMs):
  - Projection operators $P_\beta^{(a)} = \sum_{x \in \mathcal{C}} S_{0a} S_{xa}^* \mathcal{O}_x(\beta) \rightarrow$ topological charge measurements perform the *complement* of $P_\beta^{(a)}$[1]
  - Not always physical, but special cases are *symmetry protected* – we examine this

[1] More general cases exist, but are not used in this talk.

Overview

- Framework of TQC
- Introduce gapped boundaries and their framework
- Gapped boundaries for TQC
- **Universal gate set with gapped boundaries in bilayer $\nu = 1/3$ FQH**
- Summary and Outlook

Universal Gate Set

Universal (*metaplectic*) gate set for the qutrit model (Cui and Wang, 2015):

1. The single-qutrit Hadamard gate H_3, defined as $H_3|j\rangle = \frac{1}{\sqrt{3}} \sum_{i=0}^{2} \omega^{ij} |i\rangle$, $j = 0, 1, 2$, $\omega = e^{2\pi i/3}$
2. The two-qutrit entangling gate SUM_3, defined as $\mathrm{SUM}_3|i\rangle|j\rangle = |i\rangle|(i+j) \bmod 3\rangle,\ i, j = 0, 1, 2.$
3. The single-qutrit generalized phase gate $Q_3 = \mathrm{diag}(1, 1, \omega)$.
4. Any nontrivial single-qutrit classical (i.e. Clifford) gate not equal to H_3^2.
5. A projection M of a state in the qutrit space $\mathbb{C}^3$ to $\mathrm{Span}\{|0\rangle\}$ and its orthogonal complement $\mathrm{Span}\{|1\rangle, |2\rangle\}$, so that the resulting state is coherent if projected into $\mathrm{Span}\{|1\rangle, |2\rangle\}$.

Universal Gate Set - Bilayer $\nu = 1/3$ FQH

Universal (*metaplectic*) gate set for the qutrit model (Cui and Wang, 2015):

1. $H_3 = \mathcal{S}$ for $\mathcal{C} = \mathrm{SU}(3)_1$ (single layer $\nu = 1/3$ FQH), so it can be implemented by TCP.

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2. $\wedge\sigma_3^z$ can be implemented by gapped boundary braiding. Conjugate second qutrit by H_3 to get SUM_3.

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3. TCP can implement $\mathrm{diag}(1, \omega, \omega)$ (Dehn twist of $\mathrm{SU}(3)_1$). Follow by σ_3^x for Q_3.

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4. Any nontrivial single-qutrit classical (i.e. Clifford) gate not equal to H_3^2 - we have a Pauli-X from tunneling e.
5. Projective measurement - we use the TCM which is the complement of

$$P_\gamma^{(1)} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}. \tag{4}$$

Conjugating $1 - P_\gamma^{(1)}$ with the Hadamard gives the result.

Universal Gate Set - Bilayer $\nu = 1/3$ FQH

To get the projective measurement, we introduce a *symmetry-protected* topological charge measurement:

- Want to tune system s.t. quasiparticle tunneling along γ is enhanced $\rightarrow$ implement $H' = -tW_\gamma(e) + \text{h.c.}$
  - $t =$ (complex) tunneling amplitude, $W_\gamma(e) =$ tunnel-e operator

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- Implementing $M \leftrightarrow$ ground state of H' is doubly degenerate for $|e\rangle, |e^2\rangle \leftrightarrow t$ is real (beyond topological protection)
- Physically, could realize in fractional quantum spin Hall state – quantum spin Hall + time-reversal symmetry (exchange two layers)
  - Topologically equiv. to $\nu = 1/3$ Laughlin, $e =$ bound state of spin up/down quasiholes
  - e is time-reversal invariant $\rightarrow$ tunneling amplitude of e must be real $\rightarrow$ symmetry-protected TCM

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Summary

- Decoherence is a major challenge to quantum computing $\rightarrow$ **topological quantum computing** (TQC)
- TQC with anyons requires non-abelian topological phases (difficult to implement) $\rightarrow$ **engineer non-abelian objects** (e.g. **gapped boundaries**) from abelian phases
- We can get a **universal quantum computing gate set** from a purely abelian theory (bilayer $\nu = 1/3$ FQH), which is trivial for anyonic TQC
  - Topologically protected qudit encoding and Clifford gates
  - Symmetry-protected implementation for non-Clifford projection

Outlook

- Practical implementation of the symmetry-protected TCM
- More thorough study of symmetry-protected quantum computation
  - Amount of protection offered and computation power
- Other routes to engineer non-abelian objects
  - Boundary defects/parafermion zero modes from gapped boundaries of $\nu = 1/3$ FQH (Lindner et al., 2014)
  - Genons and symmetry defects (Barkeshli et al., 2014; C, Cheng, Wang, 2017)
  - How would these look when combined with gapped boundaries?

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Thanks!